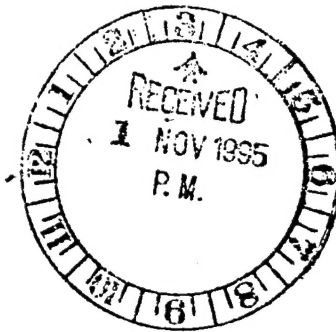


REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.</small>				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE		3. REPORT TYPE AND DATES COVERED FINAL/01 JUN 92 TO 31 MAY 95
4. TITLE AND SUBTITLE NONLINEAR ROBUST CONTROL THEORY AND APPLICATIONS			5. FUNDING NUMBERS	
6. AUTHOR(S)  JOHN C. DOYLE			3484/S5 F49620-92-J-0298	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) CALIFORNIA INSTITUTE OF TECHNOLOGY ELECTRICAL ENGINEERING PASADENA CA 91125			8. PERFORMING ORGANIZATION REPORT NUMBER AFOSR-TR- 96-0020	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NM 110 DUNCAN AVE, SUTE B115 BOLLING AFB DC 20332-0001			10. SPONSORING/MONITORING AGENCY REPORT NUMBER F49620-92-J-0298	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) Robert M'Closkey (now an Assistant Professor in the Mechanical Engineering Department at UCLA) worked in the general area of stabilization of strongly nonlinear systems. His thesis work addresses various control theoretic aspects of driftless control systems. Driftless systems area class of nonlinear problems that arise in physical systems with nonintegrable (nonholonomic) constraints and/or conservation laws. The stabilization of these systems present special problems no continuous dynamics or static function of the state can stablize the systems to a point. It is necessary to introduce explicit time variation into the control law for the continuous stabilization probe. Recently, several researchers have developed synthesis methods for generating stabilizing controllers. However, the closed-loop systems suffer from very slow convergence rates due to the fact that the control laws are Lipschitz functions.				
14. SUBJECT TERMS			15. NUMBER OF PAGES	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED			16. PRICE CODE	
18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED		19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED		20. LIMITATION OF ABSTRACT SAR(SAME AS REPORT)

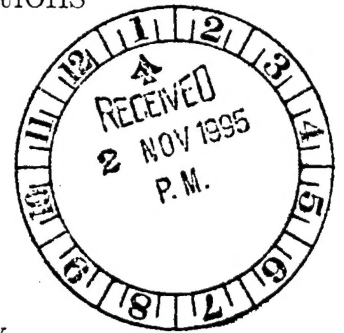
19960201 145



# Nonlinear Robust Control Theory and Applications

John C. Doyle, Principal Investigator

AFOSR AASERT Final Report  
10 October 1995  
F49620-92-J-0298



## Overview of research program for Robert T. M'Closkey

Robert M'Closkey (now an Assistant Professor in the Mechanical Engineering Department at UCLA) worked in the general area of stabilization of strongly nonlinear systems. His thesis work addresses various control theoretic aspects of driftless control systems. Driftless systems are a class of nonlinear problems that arise in physical systems with nonintegrable (non-holonomic) constraints and/or conservation laws. The stabilization of these systems present special problems: no continuous dynamic or static function of the state can stabilize the systems to a point. It is necessary to introduce explicit time variation into the control law for the continuous stabilization problem. Recently, several researchers have developed synthesis methods for generating stabilizing controllers. However, the closed-loop systems suffer from very slow convergence rates due to the fact that the control laws are Lipschitz functions.

Our work in this area dramatically improves the existing convergence rates to guarantee exponential rates for all of the state variables. The control laws remain  $C^0$  but are not Lipschitz at certain points in the phase space. In addition to exploring new design methods, we have also extended the synthesis procedures of previous authors to the mathematical framework we have proposed. Control laws designed with these tools directly address performance limiting factors such as actuator slew rates and controller effort. The theory also allows for some rudimentary robustness analysis by identifying a class of perturbations which do not locally affect the closed-loop system stability. In addition, we have constructed an experimental mobile robot that is used as a testbed to compare the performance of various controllers. The device consists of a two-wheeled car that pulls several

carts. The rolling of the wheels imposes nonholonomic constraints. The control objective is to stabilize the car/trailer system with feedback to a desired position and orientation. The experiments have demonstrated the superior performance of the exponential stabilizers over the more traditional differentiable feedbacks.

M'Closkey's work provides a first step towards generating a more comprehensive set of tools for analysis and synthesis of nonlinear control laws. As part of this work, we have developed and extended tools for analyzing stability using homogeneous functions and vector fields (relative to a non-standard dilation). These results provide a powerful set of techniques which have broad application to more general control systems. In particular, we have been able to synthesize time-varying controllers which are only  $C^0$  at the origin without losing all of the standard analysis tools which one uses to prove stability (Lyapunov functions, averaging, etc). Recent work has extended the use of these methods to strongly nonlinear systems in which a drift term is present but the use of the linearization is insufficient (or undesirable) for stabilization purposes.

## References

1. R. T. M'Closkey and R. M. Murray, "Convergence Rates for Nonholonomic Systems in Power Form", American Control Conference, 1993
  2. R. T. M'Closkey and R. M. Murray, "Nonholonomic Systems and Exponential Convergence: Some Analysis Tools", Conference on Decision and Control (CDC), 1993
  3. R. T. M'Closkey and R. M. Murray, "Experiments in Exponential Stabilization of a Mobile Robot Towing a Trailer", American Control Conference, 1994.
  4. R. T. M'Closkey and R. M. Murray, "Extending Exponential Stabilizers for Nonholonomic Systems from Kinematic Controllers to Dynamic Controllers" Symposium on Robot Control, 1994.
  5. R. T. M'Closkey and R. M. Murray "Exponential Stabilization of Driftless Nonlinear Control Systems via Time-Varying Homogeneous Feedback", Conference on Decision and Control (CDC), 1994.
- R. T. M'Closkey and R. M. Murray, "Exponential Stabilization of Driftless Nonlinear Control Systems using Homogeneous Feedback", Technical

Report CIT/CDS 95-012, California Institute of Technology. Submitted, *IEEE T. Automatic Control*.

R. M. Murray and R. T. M'Closkey, "Converting smooth, time-varying, asymptotic stabilizers for driftless systems to homogeneous, exponential stabilizers", *European Control Conference*, September 1995.